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Thermal development of radiatively active pipe flow with nonaxisymmetric circumferential convective heat loss

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Abstract—The cooling problem of the hot internal pipe flow has been investigated. Simultaneous conduction, and radiation in the fully developed inner pipe flow were considered with prescribed azimuthally varying convective heat loss at the pipe wall. A complex, nonlinear integro-differential radiative transfer equation was solved by the discrete ordinates method (or called S_N method). The energy equation was solved by control volume based finite difference technique. A parametric study was performed by varying the conduction-to-radiation parameter, optical thickness, and scattering albedo. The results have shown that initially the radiatively active medium could be more efficiently cooled down compared with the cases otherwise. But even for the case with dominant radiation, as the medium temperature was lowered, the contribution of conduction began to exceed radiation. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Forced convection combined with radiation in a thermally developing pipe flow is a long-standing topic in the high temperature heat transfer problem due to a number of diverse engineering applications such as heat exchangers in industry, solar energy collection systems, cooling processes in nuclear reactor, waste heat extraction from flue gases, and widely used manufacture of optical fiber performs called Modified Chemical Vapor Deposition [1]. In some cases the particle laden flow exists and scattering is involved. Therefore a number of studies have been performed under the various circumstances in the past several decades. The interaction of forced convection with axisymmetric radiation in a thermal entrance region of circular pipe has been investigated by many researchers [2–8]. But only a few engineers have examined the forced convection combined with axisymmetric radiation including the axial variation of radiative heat transfer [9–11]. All of these researches have been carried out under the two types of circumferential boundary conditions such as constant temperature or constant heat flux at the wall and restricted to axisymmetric situation.

Usually in high temperature heat transfer device the role of radiation is regarded as quite important, but it is a formidable problem to find exact analytical solutions due to the highly nonlinear integro-differential radiative transfer equation. Therefore, an efficient numerical tool dealing with multi-dimensional radiative heat transfer is in strong demand to analyse variously-coupled thermal problems. The discrete ordinates method, which is adopted in this study, has recently received more attention because of its efficient integration with other finite difference transport equations. This method, conceptually, belongs to a family of flux models, but corrects lack of couplings among the directional intensities present in some of conventional flux models. In the discrete ordinates method, the radiative transfer equation is solved only in a finite number of discrete directions. The principal application of discrete ordinates method has been in the field of the neutron transport [12]. The method has subsequently been applied to numerous radiative problems [13–16] with remarkable accuracy, and applied to nonaxisymmetric cylindrical enclosure [17].

As mentioned before, the pipe flow with axisymmetric radiation has been examined by many researchers. However, when the pipe is laid in crossflow of cold environment, the internal heat transfer asymmetrically occurs between internal flow and pipe wall of which outer wall is being cooled down by azimuthally varying outer-convection.

The efforts of present study are directed at solving this type of thermally developing pipe flow with nonaxisymmetric radiation by the discrete ordinates method. In order to focus on the nonaxisymmetric radiation the fluid flow is just simplified by assuming the hydrodynamically fully developed flow without solving the Navier–Stokes equation. Inside the pipe the medium thermal energy is transferred by conduction and radiation to the cold inner pipe-wall as the internal fluid flows downstream. Heat is removed at the outer pipe-wall by the azimuthally varying heat transfer coefficients or Nusselt numbers, $Nu_{\infty}(\theta)$

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NOMENCLATURE

C	thermal conductivity ratio
I	intensity
I_b	block body intensity
N	conduction to radiation parameter
n	refractive index
Nu	Nusselt number
Pe	Peclet number
q	heat flux
r	space coordinate in radial direction depicted in Fig. 1
Re	Reynolds number
T	temperature
u	velocity
z	space coordinate in axial direction depicted in Fig. 1

Greek symbols

α	thermal diffusivity
β	extinction coefficient[m^{-1}]
η, μ, ξ	directional cosines defined in equation (10)
θ	space coordinate in azimuthal direction depicted in Fig. 1
θ_Ω	directional coordinate for polar angle depicted in Fig. 1
λ	thermal conductivity
σ	Stefan–Boltzmann constant
σ_o	scattering coefficient
τ_o	optical radius
ϕ_Ω	directional coordinate for azimuthal angle depicted in Fig. 1
ω_o	scattering albedo.

Subscripts and superscripts

C	conduction
o	reference value
R	radiation
r	radial direction
T	total
w	wall
θ	azimuthal direction
∞	crossflow
$\wedge$	real quantity
	mean quantity.

which is experimentally predefined. The results are then presented by considering the effects of various parameters such as the conduction-to-radiation parameter, optical thickness, and scattering albedo.

2. ANALYSIS

As shown in Fig. 1, there is an external cross-flow over the pipe, which also provides the experimentally determined external Nusselt number variation. Inside the pipe radiatively active gray medium flows in and out with fully developed velocity. The present study, thus, deals with the interaction of asymmetric radiation with conduction for a given hydrodynamically fully developed flow without solving the momentum equation.

The radiatively active hot medium entering through one end begins to be thermally developed by internal convection/conduction and radiation in streamwise, radial and azimuthal directions. However, Sparrow and Cess [18] found that the streamwise variation of radiation can be negligible under the following conditions.

$$\frac{\bar{u}\hat{z}_o}{16\sigma n^2 \hat{T}_{in}^3 \alpha/3\beta\lambda} \gg 1. \tag{1}$$

In this study all the flow parameters are chosen to meet the above condition. Consequently, the radiation is considered to be varied only in radial and azimuthal directions, as will be shown in the following. The plot for the Nusselt numbers, $Nu_\infty(\theta)$, in Fig. 1 represents the azimuthally varying heat transfer coefficient which is applied to the circumferential boundary condition at the outer pipe wall. They are taken from the experimental data as a function of Reynolds number, Re_∞[19].

The dimensionless form of energy equation can be written as

$$u\frac{\partial T}{\partial z} = \left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right\} - \frac{\tau_o^2(1-\omega_o)}{N}\left(T^4 - \frac{1}{4\pi}\int_{\Omega=4\pi} I\,d\Omega\right) \tag{2}$$

by using the following dimensionless variables and parameters:

$$r = \hat{r}/\hat{r}_o \quad z = 2\hat{z}/\hat{r}_o Pe \quad u = \hat{u}/\bar{u} \quad \bar{u} = \int \hat{u}\,d\hat{A}/\pi\hat{r}_o^2$$
$$T = \hat{T}/\hat{T}_{in} \quad I = \hat{I}/\hat{I}_{bo} \quad q = \hat{q}\hat{r}_o/\lambda\hat{T}_{in} \quad \hat{I}_{bo} = \sigma n^2\hat{T}_{in}^4/\pi$$
$$N = \lambda\beta/4\sigma n^2\hat{T}_{in}^3 \quad \tau_o = \beta\hat{r}_o \quad \omega_o = \sigma_o/\beta \quad Pe = 2\hat{r}_o\bar{u}/\alpha \tag{3}$$

where the hat represents the dimensional variable. The inlet and boundary conditions for the energy equation with infinitely thin pipe wall, that means solid conduction being neglected at pipe wall and the same temperatures of the inner and outer pipe surfaces, are given as follows:

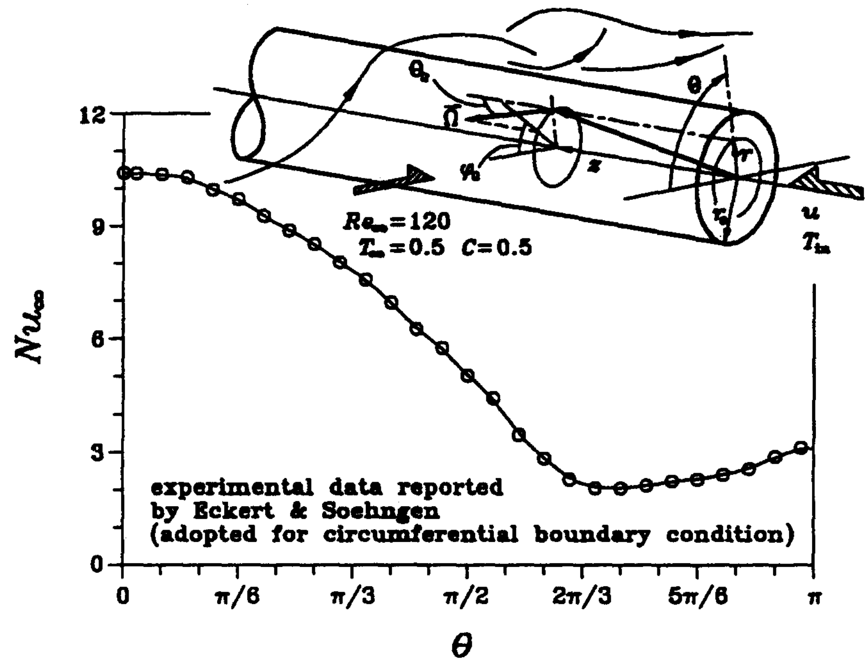

Fig. 1. Schematic of the problem and crossflow local Nusselt number for the circumferential boundary condition.

$T = 1$ at $z = 0$ finite value at $r = 0$

$$q_r^T|_w = q_r^C|_w + q_r^R|_w = \frac{C}{2} Nu_\infty (T_w - T_\infty) \quad \text{at } r = 1 \tag{4}$$

where the superscripts T, C, and R denote the total, conductive, and radiative term respectively. The Nusselt number is taken from the experimental data [19] as shown in Fig. 1. The thermal conductivity ratio, $C = \lambda_\infty/\lambda$, and dimensionless crossflow temperature, T_∞, are each set to 0.5. In the above boundary condition at $r = 1$, the periodic values are imposed on the Nusselt number following the variations in azimuthal direction in Fig. 1. The nondimensional form of radiative and conductive heat fluxes in r and θ directions can be found to be

$$\begin{Bmatrix} q_r^T \\ q_\theta^T \end{Bmatrix} = \begin{Bmatrix} q_r^C \\ q_\theta^C \end{Bmatrix} + \begin{Bmatrix} q_r^R \\ q_\theta^R \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial T}{\partial r} \\ -\dfrac{1}{r}\dfrac{\partial T}{\partial \theta} \end{Bmatrix} + \frac{\tau_o}{N}\frac{1}{4\pi}\int_{\Omega=4\pi} \begin{Bmatrix} \mu \\ \eta \end{Bmatrix} I \,\mathrm{d}\Omega. \tag{5}$$

The mixed mean temperature and the azimuthally averaged pipe wall temperature are defined at each cross-section as follows.

$$\bar{T} = \frac{\int T u \,\mathrm{d}A}{\int u \,\mathrm{d}A} \quad \bar{T}_w = \frac{\int T_w \,\mathrm{d}\theta}{2\pi}. \tag{6}$$

In order to estimate the rate of cooling of the medium along the streamwise direction, the azimuthally averaged Nusselt number is defined as

$$\overline{Nu^T} = \overline{Nu^C} + \overline{Nu^R} = \frac{\int (q_r^C|_w + q_r^R|_w)\,\mathrm{d}\theta}{\pi(\bar{T} - \bar{T}_w)}. \tag{7}$$

Based on the simplification aforementioned, the dimensionless form of the radiative transfer equation, which represents the balance of radiative energy passing in a specified direction through a small differential volume in an emitting, absorbing and scattering gray medium, can be cast as

$$\frac{1}{\tau_o}\left\{\frac{\mu}{r}\frac{\partial (rI)}{\partial r} - \frac{1}{r}\frac{\partial (\eta I)}{\partial \varphi_\Omega} + \frac{\eta}{r}\frac{\partial I}{\partial \theta}\right\} + I = (1-\omega_o)T^4 + \frac{\omega_o}{4\pi}\int_{\Omega'=4\pi} I' \,\mathrm{d}\Omega' \tag{8}$$

where the primed values represent the incoming direction and the corresponding direction cosines are

$$\mu = \sin\theta_\Omega \cos\varphi_\Omega \quad \eta = \sin\theta_\Omega \sin\varphi_\Omega \quad \xi = \cos\theta_\Omega. \tag{9}$$

In the following study only isotropic scattering is considered. Boundary conditions for the radiative transfer equation at the centre axis and the inner pipe wall assumed to be black body are respectively given by

$$I = I' \quad \text{with } \vec{\Omega} = -\vec{\Omega}' (\text{for } \mu > 0) \quad \text{at } r = 0$$

$$I = T_w^4 (\text{for } \mu < 0) \quad \text{at } r = 1. \tag{10}$$

In this study the radiative transfer equation (8) is solved by the discrete ordinates method. Since all the details about this method are well described in the previous paper [16], they are not repeated here for its brevity. By this method the intensity is only solved in a finite number of directions spanning a full range of the total solid angle, 4π. Preliminary evaluations revealed that S_4 approximation is quite adequate in the present work, for no measurable gain was obtained in accuracy by higher order approximations such as S_6 and S_8. Therefore S_4 approximation is used in this study. The ordinate directions and quadratic weighting factors adopted here are also explained in ref. [16]. The energy equation (2) is discretized by the finite difference technique based on the control volume approach and solved by tridiagonal-matrix algorithm. To validate its symmetry about the pipe mid-plane the full cross-section is solved from $\theta = 0$ to 2π. The iteration is repeated until convergence is accomplished. A 17×56 uniform grid system is adopted at each cross section, since a finer grid did not yield much difference. The convergence is monitored through temperature difference between two iteration steps.

3. RESULTS AND DISCUSSION

To investigate the effect of radiation on the thermally developing pipe flow with prescribed circumferentially varying convective heat loss, the numerical calculation has been performed for no radiation and four combinations of such three radiation involving parameters as the conduction-to-radiation parameter, N, the optical radius, τ_0 and scattering albedo, ω_0; the first one for $N = 0.1$, $\tau_0 = 1$, and $\omega_0 = 0.1$; the second one for $N = 0.02$, $\tau_0 = 1$, and $\omega_0 = 0.1$ to seek the effect of conduction-to-radiation parameter and represented as the dashed line; the third one for $N = 0.1$, $\tau_0 = 5$ and $\omega_0 = 0.1$ to examine the effect of optical radius; the fourth one for $N = 0.1$, $\tau_0 = 1$ and $\omega_0 = 0.8$ to investigate the effect of scattering albedo.

Figure 2 shows the isotherm variation for half domain as the flow is being developed along downstream. The external flow direction over the pipe is from left to right as shown in the figure. Owing to this crossline, the temperature distribution is observed to be nonaxisymmetric and the hot medium always rapidly cools down near stagnation point as seen in all pipe cross-sections, which leads to a steep temperature gradient therein. The high temperature zone is at the inner core. As the hot medium flows through the pipe, eventually the medium temperature reaches that of the environment. From then on no further heat loss to outside is to occur.

The effect of conduction-to-radiation parameter, N, can be figured out by comparing the cases (a) and (b). Since the radiation plays a more significant role as N decreases, the fluid more rapidly cools down due to its salient deep-penetrating effect of radiation. Consequently, the medium temperature becomes more uniform for smaller N and thus the conductive heat flux would be smaller. By comparison of case (a) with (c) in Fig. 2, the effect of optical radius τ_0 can be examined. It is seen that the rate of cooling of the medium becomes faster as the optical radius increases from 1 to 5. This results from the fact that the more radiatively active the flow medium, the larger its total emission becomes. The effect of scattering is illustrated in Fig. 2(d). The scattering of the medium reduces the capability of emitting energy and thus less energy is directed towards the wall boundary. This results in diminishing the cooling rate of the medium when ω_0 is increased from 0.1 to 0.8.

The detailed cooling history of the medium is graphically represented in Fig. 3, in which the left-hand side one represents the wall temperature variation as the azimuthal angle increases from 0 to π. The radial temperature distribution in the range of $0 < r < 1$ is plotted for two azimuthal angles ($\theta = 0$ and π) on the right-hand side in the figure. By comparing the contours at $z = 1.601$, it is noticeable that the whole medium for the case (b) is being cooled faster than the others. But at near entrance region ($z = 0.03$ or 0.186) the temperature gradient at the pipe wall for the case (b) is found to be rather small. This can be explained as follows. If the conduction is dominant over the radiation, very steep temperature gradient is developed near the pipe wall. Therefore the medium is mainly cooled by the conduction mode. Reversely, when the radiation becomes dominant, the temperature gradient at the pipe wall is small. Therefore, the conductive heat transfer to the wall is small. However, the whole medium is primarily cooled down by the direct emission of the medium to the wall. In other words the emission of the medium is the primarily factor to determine the cooling rate, once the radiation is involved. As a result of this composite phenomenon, the temperature at the wall is higher for radiatively active medium in the entrance region.

To quantify the cooling rate along the streamwise direction of the pipe, the mixed mean temperature and the averaged wall temperature defined in equation (6) are plotted in Fig. 4. The streamwise development of the averaged temperatures is slowest for the case without radiation. Especially the mixed mean temperature contour shows that the medium cools down faster as the radiation becomes more dominant. However, the averaged wall temperature has inverse tendency during the initial thermally developing period due to the strong influence of conduction near the wall. Consequently, there seems an inverse correlation between two types of temperature curves from the point view of medium cooling. As the flow cools down more the conduction incurred by temperature gradient is smaller and the inverse nature of those two averaged temperatures disappears. In all cases the medium temperature finally converges to the surrounding temperature 0.5 for $z > 3$.

In Fig. 5 the local heat fluxes transferred to the pipe wall are represented at four axial positions ($z = 0.03$, 0.186, 0.705 and 1.601) on the logarithmic scale vs the azimuthal angle. In overall, the conductive and radiative heat transfers are of the same order. The heat loss reaches a maximum at the stagnation point

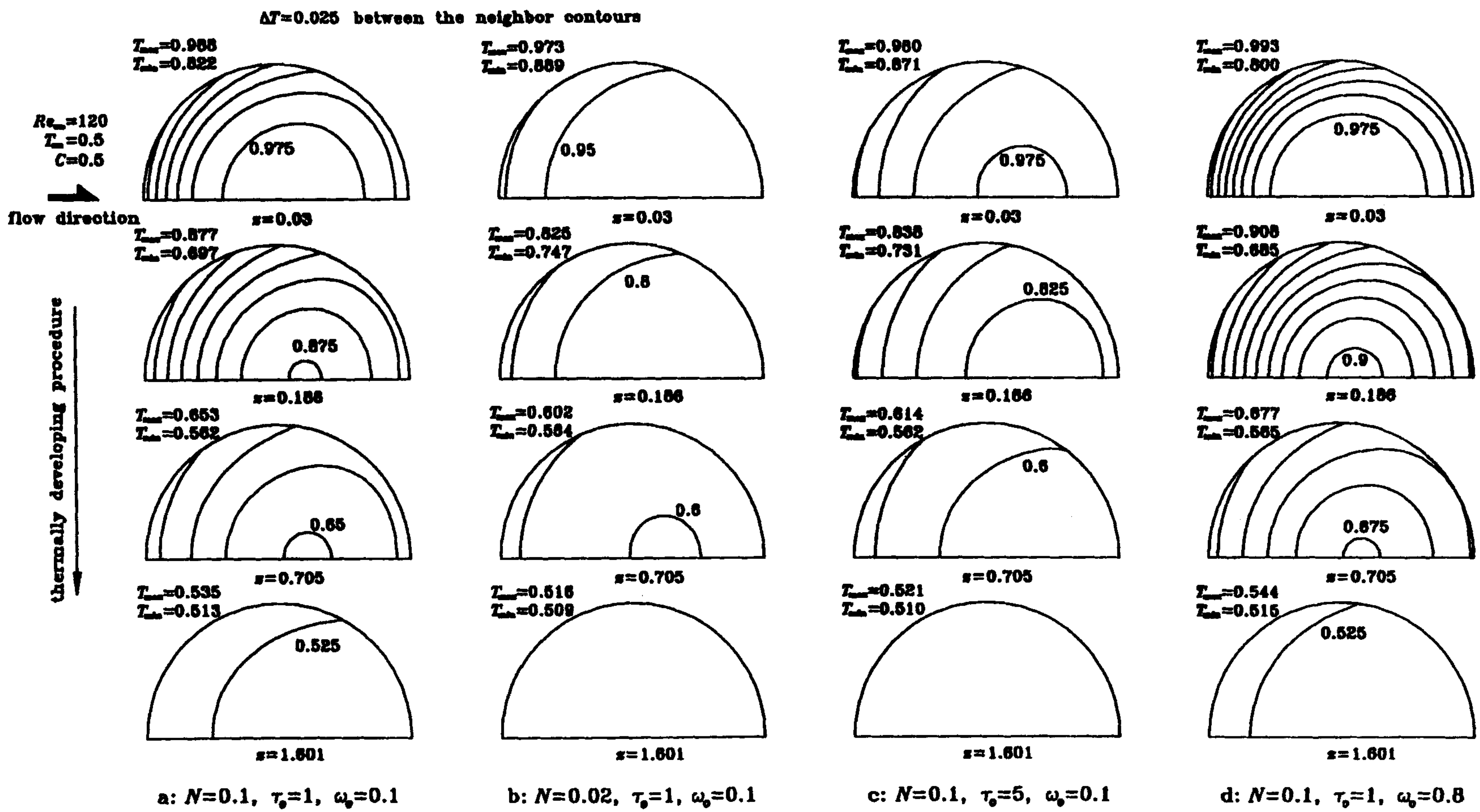

Fig. 2. Thermally developing isothermal contours for: (a) $N = 0.1$, $\tau_o = 1$, $\omega_o = 0.1$; (b) $N = 0.02$, $\tau_o = 1$, $\omega_o = 0.1$; (c) $N = 0.1$, $\tau_o = 5$, $\omega_o = 0.1$ and (d) $N = 0.1$, $\tau_o = 1$, $\omega_o = 0.8$.

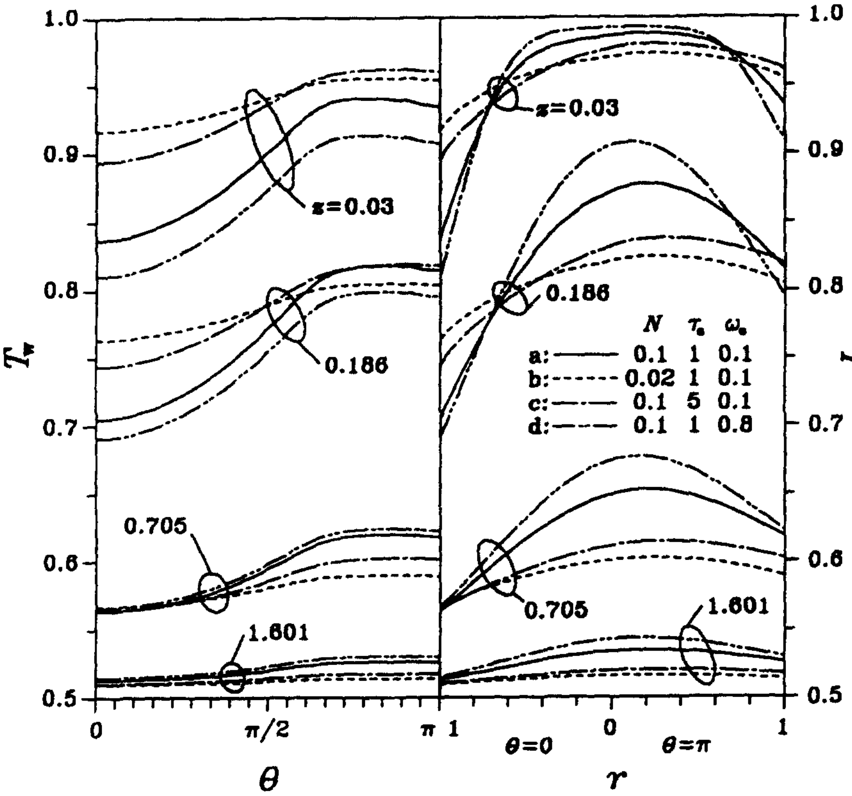

Fig. 3. Azimuthal variation of the wall temperature and radial temperature variation at $\theta = 0$ and π.

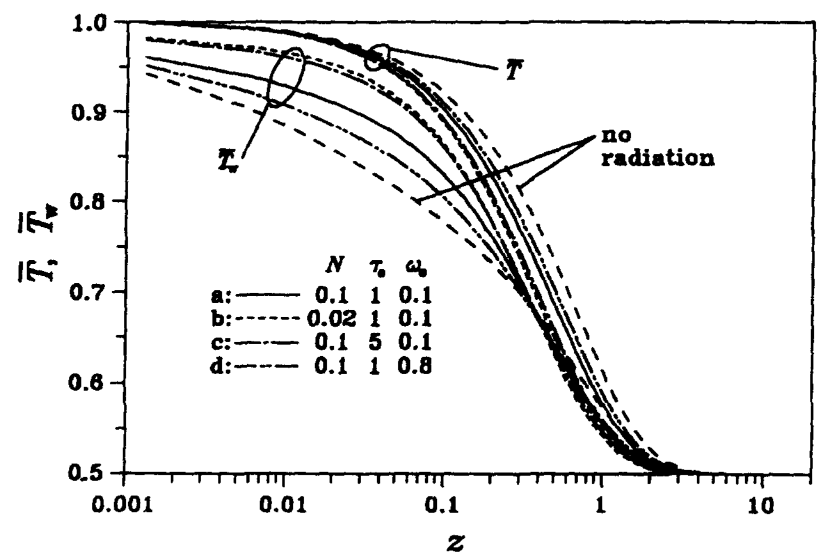

Fig. 4. Streamwise variation of mixed mean temperature and azimuthally averaged wall temperature.

$\theta = 0$ and then decreases as θ increases following the crossflow Nusselt number profile. In the rear wake region it again slightly increases. For both cases, (b) (smaller conduction-to-radiation parameter) and (c) (larger optical radius), the radiative heat flux is observed to be relatively bigger than the conductive heat flux in the upstream. Therefore, the more radiatively active the medium, the more heat loss there occurs at the pipe wall. After some distance from entrance where intense cooling takes place, the radiative heat loss begins to be reserved, e.g. $z = 0.705$. But this is due to the fact that the medium already cooled down enough emits less radiative energy.

The axial variations of total Nusselt number, $\overline{Nu}^{T}$, defined in equation (7) and the ratio of conductive portion to total heat flux to the pipe wall, $\overline{Nu}^{C}/\overline{Nu}^{T}$,

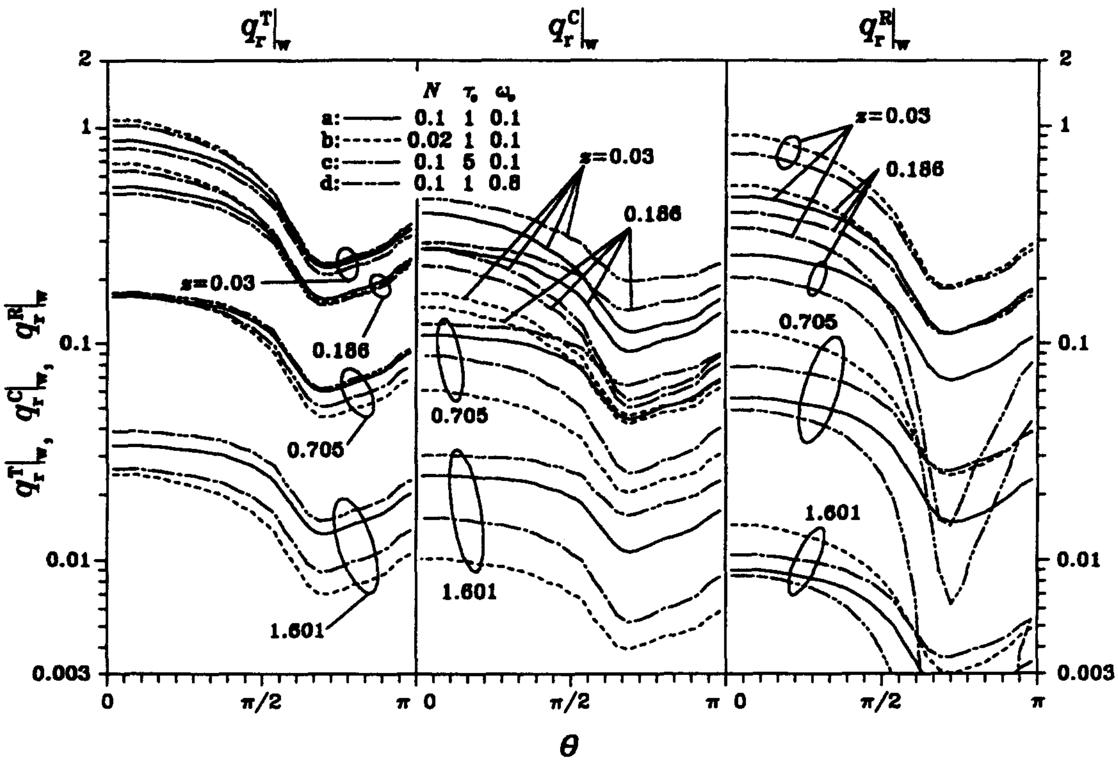

Fig. 5. Azimuthal variation of wall total as well as conductive heat fluxes at four different axial positions.

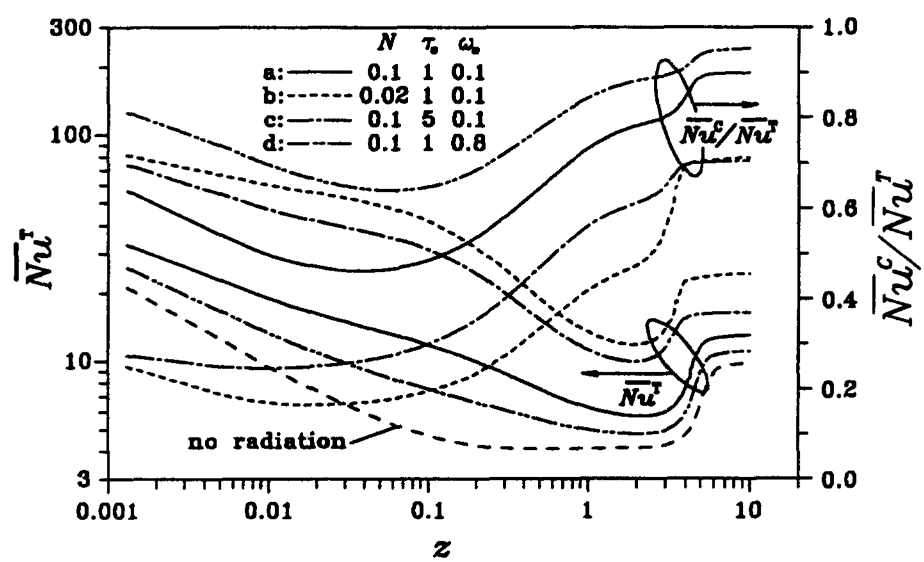

Fig. 6. Streamwise variation of the total mean Nusselt number and the ratio of conductive to total wall heat flux.

are represented in Fig. 6. The total Nusselt number becomes higher for the case (b) in which the radiation is dominant and has the lowest value for no radiation. It decreases as cooling continues. As the temperature difference in the denominator of equation (7) is reduced, its rate of decrease overwhelms the decreasing rate of heat loss. That is why the total Nusselt number increases again and finally reaches a plateau.

As can be seen in the plot of $\overline{Nu^C}/\overline{Nu^T}$, the contribution of radiation in the cooling is maximum for case (b) and minimum for case (d). In general $\overline{Nu^C}/\overline{Nu^T}$ decreases as the steep temperature gradient near the wall region is reduced. After some distance, it starts to increase again since the emission of the medium is decreased due to its low temperature. For the cases of (b) and (c), the initial contribution of conduction is less than 20% in entrance region and then attains 70%. The more scattering as for the case (d) makes the emitting energy less available. Therefore, the conductive contribution is biggest.

4. CONCLUSIONS

Combined conductive and radiative heat transfer with a given hydrodynamically fully developed inner pipe flow has been investigated. The outer convective heat exchange with external crossflow was pre-described. The radiative transfer equation was numerically solved by using the discrete ordinates method, of which accuracy and efficiency were well verified. Radial and azimuthal variations of radiation were taken into account.

By performing the parametric study using different values of such parameters as conduction-to-radiation parameter, optical radius, and scattering albedo, the results show that the internal medium is more efficiently being cooled down when the radiation mode becomes predominant. In other words, the larger optical radius or the smaller conduction-to-radiation parameter makes the medium a more efficient emitter. And thus, the medium temperature is lowered faster. Even if the initial radiative contribution is higher, its role is diminished after all since the medium temperature decreases along downstream. Consequently, initially the more radiatively active medium is being cooled down more efficiently than otherwise. However, as the medium temperature is lowered, the conduction comes to play a more significant role.

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